

“Hybrid Control Systems”

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Project: Scanning Electron Microscopy

Introduction

In Scanning Electron Microscopy (SEM) a (bundled) beam of electrons, guided by an electromagnetic lens system, hits a specimen target (sample). This excites the specimen which can be detected. The specimen may e.g. reflect the electron, can release secondary electrons or x-rays may be induced as a result. This varies with the material the specimen consists of. Various detectors can be used to measure the effect. A schematic representation of an SEM is depicted in Figure 1.

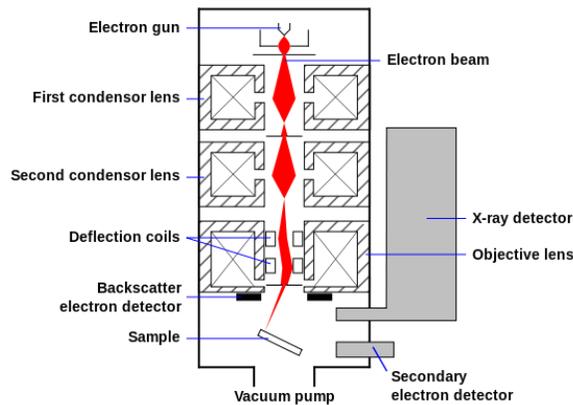


Figure 1: Schematic of an SEM [source: http://en.wikipedia.org/wiki/File:Schema_MEB_%28en%29.svg]

The relation between the original energy of the beam and the effect at the detector level can be used to identify the material of the specimen at a specific spot. This focus spot is where the electron beam is bundled to a small diameter. The identification is done by integrating the sensor value, accumulating the detector energy, over a specific time, the pixel dwell time. This gives a pixel value in the image.

In SEM the focus spot is moved over the area that is of interest of specimen in a (raster) pattern. This raster pattern is often a line by line sweep. The specimen area is gridded, such that each grid area corresponds to a pixel in the image. While the focus spot is in a specific area of the grid, the detector value will contribute to the value of the corresponding pixel.

During a line sweep, the focus spot flows over the specimen. When it reaches the edge of the area that is of interest, it jumps to the start of the next line. When it reaches the end of the last line it

returns to the start of the first line. (Note that, in general, lines have an equal number of pixels, but is not necessarily the case)

Note: In general the focus spot is not an ideal point, but is a distribution around a center point. This allows for an area of the specimen to be excited, the result of this excitation can be detected in various ways.

Assumptions

Ideal focus spot: the location of the focus spot on the specimen is represented by a point on the specimen. (This spot could correspond to the center of the beam distribution)

Ideal bundle size: if the beam sweeps in a straight line from middle of the left edge of the pixel area to the right, it will result in a value that is sufficiently representative of the specimen material in the pixel area.

Predetermined scan area: bounds for the focus spot are known

System model

A schematic of the system is show in in Figure 2.

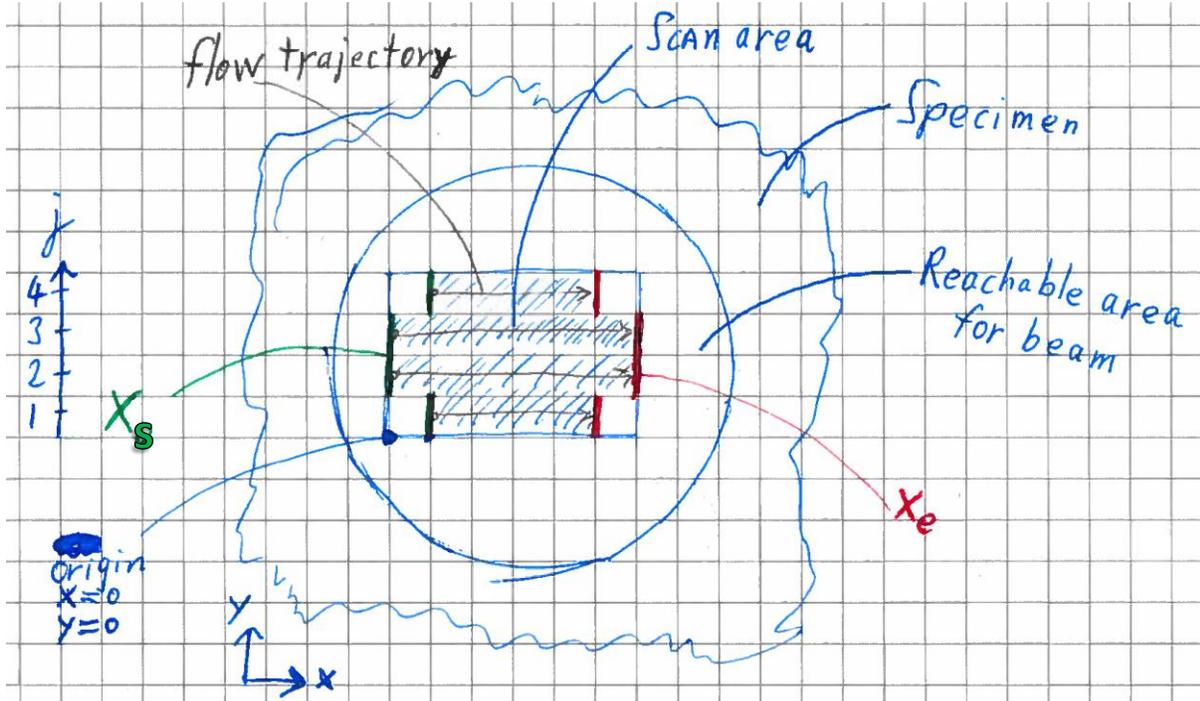


Figure 2: Scanning of a specimen in an SEM

The following hybrid model $H = (C_p, F_p, D_p, G_p, *)$ represents the movement of the focus spot with position (x, y) over the specimen, using a line-by-line raster pattern with non-consistent number of pixels per line.

$$F_p(z, u) = \begin{cases} \dot{x} = v_{sweep} \\ \dot{y} = 0 \\ \dot{j} = 0 \end{cases}, \quad G_p(z, u) = \begin{cases} x^+ = \begin{cases} x_s(j+1) & j < n \\ x_s(1) & j = n \end{cases} \\ y^+ = \begin{cases} y + d_{pixel} & j < n \\ d_{pixel}/2 & j = n \end{cases} \\ j^+ = \begin{cases} j+1 & j < n \\ 1 & j = n \end{cases} \end{cases}, \quad z = \begin{bmatrix} x \\ y \\ j \end{bmatrix}, \quad u = \begin{bmatrix} v_{sweep} \\ d_{pixel} \\ n \\ x_s \\ x_e \end{bmatrix}$$

$$C_p(z, u) := \{(z, u) \mid x_s(k) \leq x \leq x_e(k), j = k, k \in \{1, 2, \dots, n\}\}$$

$$D_p(z, u) := \{(z, u) \mid x = x_e(k), j = k, k \in \{1, 2, \dots, n\}\}$$

Where $d_{pixel} > 0$, $\frac{d_{pixel}}{\tau_{dwell}} = v_{sweep} > 0$, and $n \geq 1$ are the pixel width/height (taken equal), the sweep speed, and the number of lines in the raster, respectively, which are given constants. The bounds

(edges of the raster) for the focus spot, x_s and x_e , are the start and end positions for each line, respectively. Each element of the array corresponds to a line in the raster, where we have that $0 \leq x_0(k) < x_e(k)$ for each k , such that the scan position lies in the positive quadrant. Note that, for a well-defined grid, $\frac{x_e(k) - x_0(k)}{d_{pixel}} = m_k$ is an integer value representing the number of pixels in line k .

The system is a hybrid system since state x has nontrivial flow (continuous) and jump (discrete) dynamics. During a line sweep the system flows in positive x direction. At the end of the line the system jumps to the start of the next line.

Note: A pixel counter (discrete dynamics) could be added, but it is not very interesting in this project since it does not add hybrid dynamics (only discrete).

Note: The scanning raster could be rotated, such that the sweep is both in x and y -direction. This can be solved by a coordinate transformation.

Extensions:

- Varying sweep speeds per pixel or line could be used.
- The system could jump at the edge of any pixel and to any new pixel edge.
- Disturbances may affect the x and y position of the beam during flow.
- Disturbances may affect the jump map for the x and y position of the beam.
- Modeling the set point control system that controls the x and y position of the beam as a negative feedback on the error in the (x,y) -position.

Simulations

The scans will normally start at the beginning of a line. The system is initialized with initial state

$$z_0 = (x_0, y_0, j_0) = \left(x_s(k), d_{pixel} \left/ \vphantom{x_s(k)} \right. 2 + d_{pixel} \cdot (k-1), k \right) \text{ for some integer } 1 \leq k \leq n.$$

If the input u is kept constant, then the solutions of the system, the hybrid arcs, are non-trivial, complete and compact. Also they are periodic, since the system will return to the initial state after a fixed period of time, which depends on the inputs and the scan raster.

Simulations of the system for initial condition $z_0 = (x_0, y_0, j_0) = \left(x_s(1), d_{pixel} \left/ \vphantom{x_s(1)} \right. 2, 1 \right)$ are shown in

Figure 3, Figure 4 and Figure 5. Parameters chosen for the simulation are

$$\left. \begin{array}{l} d_{pixel} = 1 \\ \tau_{dwell} = 0.2 \end{array} \right\} \Rightarrow v_{sweep} = 5 \text{ and a curved pattern for the scanning raster: 7 lines by 12 columns with}$$

$$n = 7$$

the three corner pixels removed. The simulation horizon is taken to be 15 seconds or 10 jumps.

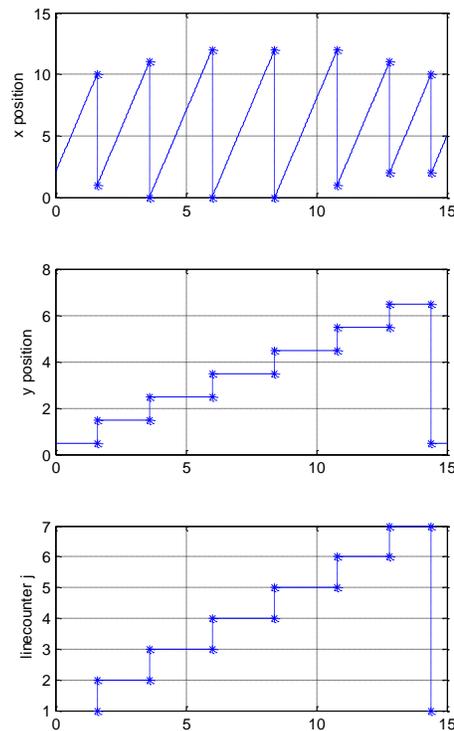


Figure 3: Trajectories of the states of the system

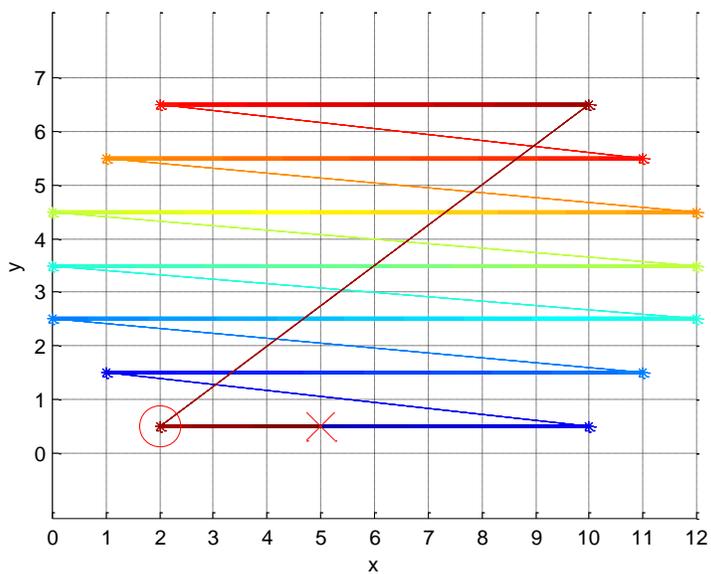


Figure 4: Position of the focus point of the SEM, starting at the red circle in dark blue and ending at the red cross in dark red after just over one scan period. The grid squares that have a thick line going through them are the pixels that are scanned.

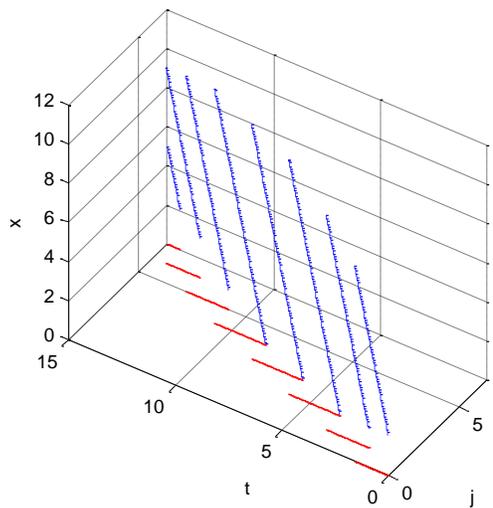


Figure 5: Hybrid arc of the x-position of the beam.

Analysis

The system H satisfies the hybrid basic conditions (HBCs):

- I. C and D are closed subsets of \mathbb{R}^n : C is a closed line and D is a point in x , for each line number j . The number of lines n is discrete and finite. The union of a finite number of closed sets is finite.
- II. F is single valued, F is continuous and C is in the domain of F .
- III. G is single valued, G is continuous and D is in the domain of G .

The trajectories of the system are periodic, this is inherent to the repetitive nature of the scanning procedure.

No feedback of position errors is implemented. Therefore disturbances and uncertainties in the dynamics will produce errors that will propagate in the trajectories of the system, giving a perturbed system H_δ . The beam position may go out of the area of interest and produce poor scan results.

Position error feedback during flow or position reset during jumps could cope with this to minimize the scan error. The feedback could make perturbed trajectories converge to the nominal trajectory that is modeled and simulated in this document. The nominal trajectory is then a compact set A , that is stable and locally pre-attractive, for the system H_δ . This implies robustness if H_δ satisfies the HBCs if a well-designed feedback controller is implemented for the position error. Note that in practice it is very difficult to measure the beam position without affecting the beam itself.

Conclusion

In this report it is shown that the scanning of a region of interest of a specimen in Scanning Electron Microscopy can be modeled as a hybrid dynamical system. The sweeping of the electron beam over the specimen can be described by a flow map, the scan area is described by a flow set, the movement of the beam to the start of a new line is described by a jump map, and the end of a line of pixels is described by a jump set. Simulations show the trajectories of the system, the hybrid arc, with flows and jumps. A feedback method for the error in the beam position, yielding robustness for the case where disturbances or uncertainties are affecting the system, is described. Suggestions are made for useful extensions to the model, which lie beyond the scope of this document.